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UPWARDS FIRED BULLET TERMINAL VELOCITY

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A numerical study is carried out to investigate a 7.62 mm upwards fired bullet flight at the trajectory apex and falling after that. The bullet model includes an aerodynamic model which basically covers angles of attack up to 180º. Computational fluid dynamics is utilized to estimate the aerodynamic properties at the high angles of attack. The role of Magnus-phenomena at the apex and in the descent is particularly studied. A buffeting-type phenomenon, new to the present authors in this particular context was detected in the simulations. In case of bullet-flow frequency matching the bullet fast mode oscillation is seen to grow which may retard falling velocity at the late part of descending flight phase. The simulated bullet terminal velocities were compared with scarce experimental data available and the agreement was found satisfactory.

INTRODUCTION

In this paper, a computational model for a 7.62 mm bullet was created and six degrees of freedom (6-dof) simulations were undertaken to find out some possible trajectories for upwards fired bullets. The bullet aerodynamic model and initial trajectory angle were varied in the simulations and the flight at the apex and after it was studied to estimate some model time dependencies effect on the terminal velocity. Also some frequency-domain analysis was utilized to find out the bullet oscillation mode characteristics at crucial trajectory points. Finally, the falling bullet effect on life was shortly estimated based on the literature.

BACKGROUND OF THE STUDY

The motivation of the ongoing research work is to find out a simple modification for typical bullet geometry to decrease the falling terminal velocity possibly through entire launch angle region. However, actual limited range training bullet with excessive geometry modifications is not looked for. The flight of ordinary bullet geometry is examined at first in this study in order to gain knowledge of the phenomena present.

One way to gain the bullet subsonic instability wished might be to utilize the Magnus-phenomena. This aerodynamic interaction between the pitch- and yaw-levels is due to bullet spin and flow viscous phenomena. Because of it the total aerodynamic moment vector will not remain oblique to the level defined by bullet symmetry axis and velocity vector. This may cause bullet rapid dynamical instability with a low flight velocity as a consequence.

Some aerodynamic moment is needed to evoke fast spinning bullet turning and make the bullet centre line to follow the velocity vector. It turned out in the simulations of Ref. [1] that the Magnus-moment is an important factor for projectile flight at the apex and after that when fired about straight upwards. Generally the larger the high yaw angle positive Magnus-moment value is the easier bullet turns nose down at the apex [1]. Positive Magnus-moment is defined here to turn the projectile nose to the direction of the normal coning motion which is clockwise seen from behind (in case the projectile is also spinning clockwise).

BULLET AERODYNAMICS

The 7.62 mm bullet geometry studied is described closely in section "bullet geometry". The bullet aerodynamic properties were estimated at first using a simplified engineering method of Ref. [3] (see also Ref. [1]). Some high angle of attack published data was studied [4-6] in order to end up to at least qualitatively correct model for the geometry studied. Also some more theoretically oriented papers from this field were explored [7-9] to assess applicability of the results obtained in this study.

Besides the engineering method and literature, two CFD software packages were used in the simulations: open source and free OpenFOAM 1.7.0 [10] and commercial ANSYS Fluent 12.1 [11]. The ANSYS Fluent licenses were provided by CSC - IT Center for Science Ltd.

The bullet diameter based Reynolds numbers are subcritical (24 000 and 3 600) here and a laminar separation was expected to occur at high angles of attack from both sides of the bullet [5]. The rifle caused bullet surface groove effects were not considered in this study.

At first the flow field was simulated at the flight altitude 1 000 meters where the bullet falling velocity was taken to be 50 m/s and the spin rate was 1 000 Hz. The corresponding dimensionless spin (or spin ratio) $\hat{p}$ is 0.479. The second flow-simulation case with only the velocity value changed (7.5 m/s, spin ratio 3.19) was chosen based on the results obtained. Other free stream flow parameters were not varied and are listed in table 1. The CFD- simulations were conducted at four different angles of attack: 45º, 90º, 110º and 135º.

TABLE I. FREE-STREAM FLOW PARAMETERS AND REFERENCE DIMENSIONS.

Characteristics	Value	Characteristics	Value
Velocity, V	50 m/s	Reference length, d	7.62·10-3 m
Pressure, p	89 875 Pa	Reference area, S	4.56·10-5 m2
Density, ρ	1.1116 kg/m3	Reynolds number, Re_d	24 000 and 3 600
Dynamic viscosity, μ	17.58·10-6 kg/ms	Spin rate	6 283 rad/s
Temperature, T	281.65 K		(1 000 rps)

The free stream flow velocity is strictly subsonic and incompressible (Ma = 0.15 and Ma = 0.0225). Incompressible pressure-based flow solvers were used in OpenFOAM, but density-based compressible flow solver was used in Fluent.

The same computational grid (see Figure 1) was used with both OpenFOAM and ANSYS Fluent.

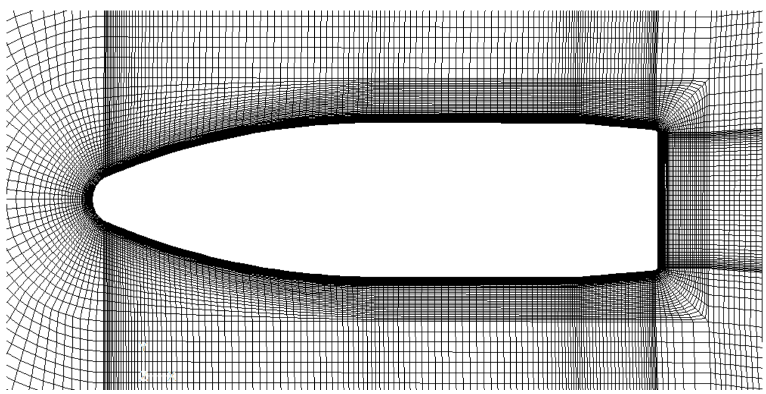

Figure 1. The computational grid near the 7.62 mm bullet surface. The grid extends approximately 0.6 meters to all directions.

The flow-field obtained at the end of the transient simulation at the yaw angle 135º is depicted in Figure 2. Figure illustrates streamlines near the bullet body making the flow separation line discernible. The line is obscure near the bullet base where the flow is fluctuating periodically.

The Magnus-moment coefficient time history obtained at angle of attack 135º is depicted in Figure 3. The bullet velocity is 50 m/s and the result is given in the CFD-coordinate system with positive direction “nose to left” (looking forward from the back of the bullet). The coefficient is seen to oscillate at frequencies between 1 000 - 1 500 Hz.

The average mean flow oscillation frequency value is approximately 1 250 Hz. The corresponding wake instability Strouhal ($St = fd/V$) number is 0.19 which is the typical value for a cylinder in the Reynolds number region from 300 at least up to about 200 000.

The large-scale flow time-dependent behavior at high angles of attack (i.e. Von Karman vortex -street) is now believed to be captured properly enough with the simulation approach used.

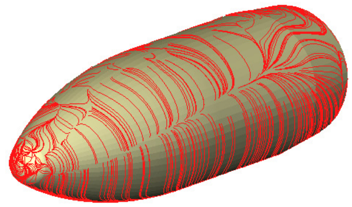

Figure 2. Streamlines near the bullet body at the yaw angle 135º (V=50 m/s and $\hat{p}$ =0.479).

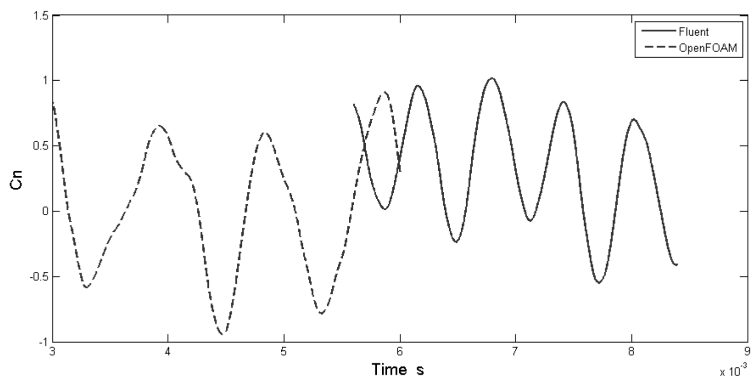

Figure 3. The 7.62 mm projectile Yaw moment coefficient C_n versus time (V=50 m/s and $\hat{p}$ =0.479) at 135º angles of attack.

The Strouhal number was used to estimate the flow velocity for some possible bullet/flow resonance to appear (at about 170 - 180 Hz based on the bullet Eigenvalues solved). The velocity 7.5 m/s was chosen for the second phase flow simulations. The Fluent-based Magnus-moment coefficient oscillation frequency obtained at the yaw angles 90º and 135º was about 200 Hz (at 7.5 m/s) and the resonance is supposed to be possible at small flight velocities.

Aerodynamic Model

The only aerodynamic coefficient of the model discussed in this chapter is the Magnus-moment coefficient. The bullet aerodynamic model is given as schematic closed-form formulas in Appendix A to facilitate easy repetition of simulations.

The steady Magnus-moment coefficient C_n (at $\hat{p}$ =1) model used in the trajectory simulations at small velocities is depicted in Figure 4 as a function of yaw angle. The moment coefficient is given in the coordinate system used in the trajectory simulations (positive nose to right about the mass center). The overall behavior is based on the engineering method results, literature and CFD.

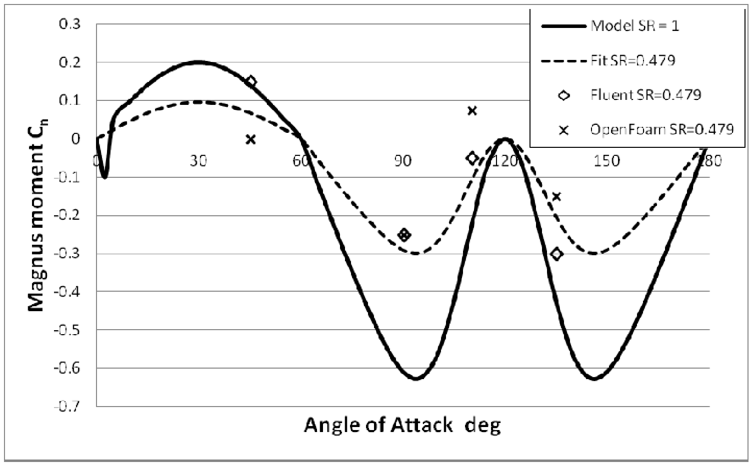

Figure 4. The 7.62 mm projectile Magnus-moment coefficient $C_n f(\alpha)$-model used in the trajectory simulations ($\hat{p}$ =1). The high velocity moment behavior is similar without the reverse effect at small yaw angles. The moment coefficient value depends linearly on the dimensionless spin value.

Experimentally obtained and CFD-based results published show negative Magnus-moment coefficient values at small angles of attack at subsonic and transonic region (see Fig 4 at left, the phenomena is included based on [12][13], not simulated in this study).

The Magnus-moment coefficient sinusoidal oscillation amplitude was in the trajectory simulations made to depend on the yaw angle only. The oscillation values used were ±0 (60º), ±0.3 (90º), ±0.3 (110º), ±0.6 (135º) and ±0 (α > 175º, unknown, not simulated). The coefficient oscillation frequency applied was 175 Hz in the velocity region 0…10 m/s near the apex.

BULLET GEOMETRY

The trajectory simulations were carried out to 7.62 mm 9.5 g generic bullet which was fired upwards with initial velocity 850 m/s. The bullet data is given in Table 2 and the geometry schematics is shown in Figure 5. The weapon rifle makes one spin while the bullet travels 304.8 mm (12 inches) resulting to the initial spin value 3 150 rounds/s.

TABLE II. PHYSICAL AND GEOMETRICAL DATA USED FOR THE 7.62 MM BULLET.

Characteristics	Value
Diameter, d	7.62 mm
Weight, m	9.5 g
Length, l	28 mm
Center of gravity (CG)	17 mm (from the nose)
Nose length, l_n	14 mm
Moment of inertia, I_x	$6 \cdot 10^{-8}$ kgm^2 (longitudinal)
Moment of inertia, I_y=I_z	$4 \cdot 10^{-7}$ kgm^2 (transverse)

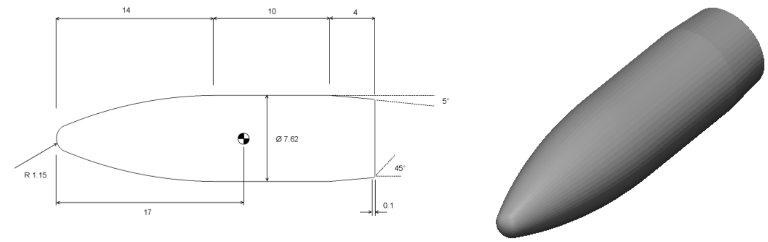

Figure 5. The 7.62 mm bullet geometry studied.

TRAJECTORY SIMULATION MODEL

Two different 6-dof simulation codes were written in order to simulate the bullet flight. The mathematical model needed to accurately enough capture the phenomena is described in many text books (see for example Ref. [14]).

The projectile body-fixed and earth-fixed coordinate systems used are depicted in Fig 6. The projectile body-fixed coordinate system was defined in two different ways (spinning and non-spinning) in two separate simulation codes used. The trajectories were integrated numerically (RK4) and the atmosphere model used was the ISO standard one.

The bullet natural oscillation modes were also solved during the flight path evaluation (see also [16]). The analysis was carried out to find out the possible resonance phenomenon of the bullet and flow. The bullet fast mode natural frequency was found to be about 170-180 Hz near the apex. The result is nearby the very small flight velocity CFD-based mean flow Von Karman vortex -street oscillation obtained. Some oscillatory coupling was then expected to appear there either at the end of ascending or at the beginning descending flight part. In the present paper it is assumed that the bullet/flow time-dependent interactions are buffeting type phenomena without bullet response caused effects on the flow.

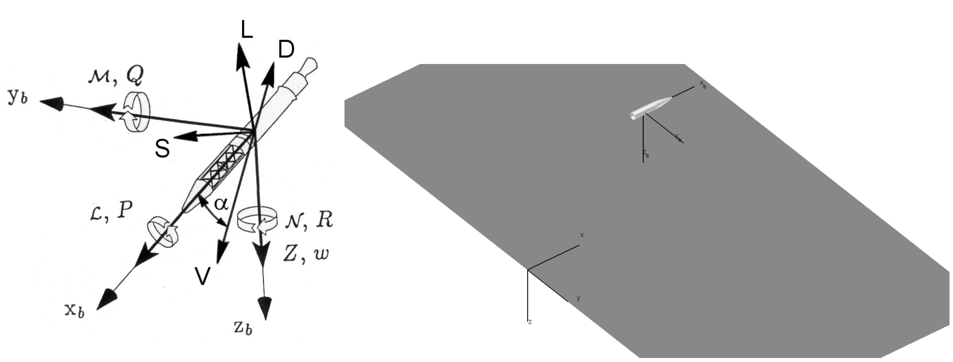

Figure 6. The coordinate systems used. The positive moments and angular velocities are also depicted. The total angle of attack α is the angle between the x_b-axis and the velocity vector V.

TRAJECTORY COMPUTATION RESULTS AND DISCUSSION

The bullet turning nose down takes place up to about launch angle 80º. The aerodynamic model time-dependencies studied had no effect on the largest bullet nose-down turning launch angle value since no matching of the natural frequencies (flow/bullet) occurred.

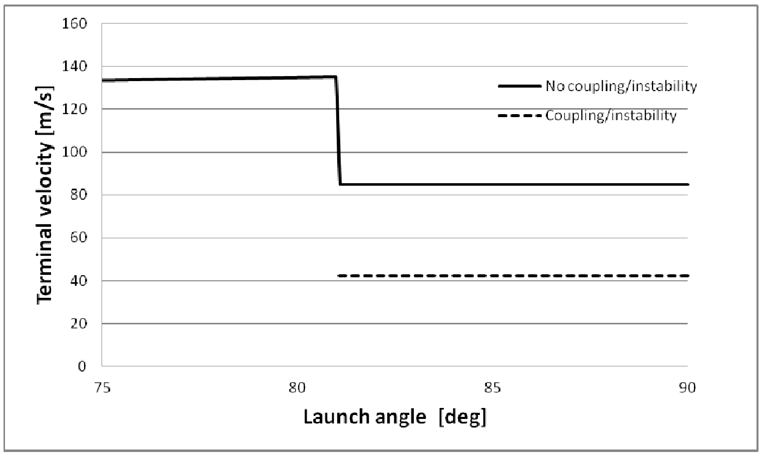

Figure 7. The bullet terminal velocity versus launch angle.

The simulated bullet terminal velocities (TV) are depicted in Figure 7 as a function of launch elevation angle. The launch angle zone shown is the one where the most interesting phenomena were believed to occur in this study. The trajectory apex is at about 3 000 meters altitude in the region.

The terminal velocities obtained were 100…135 m/s in the launch angle region 15º…80º respectively. However, at this point of research work the Magnus-effects for the nose first falling bullet are probably not modeled adequately and this is a subject of further studies. At higher launch angles the bullet lands either base first with velocity approx. 85 m/s or more or less sideways with lower velocity (min. value obtained about 40 m/s).

After failing to turn with the velocity vector (at launch angles above 80º) the bullet ends up to fall at about yaw angle 180º. The bullet descending part in-flight behavior is determined ia by disturbances present and the Magnus-moment coefficient slope $f(\alpha)$ at and near the yaw angle 180º. The slope in this study is now determined by the fit used for Magnus-moment and is somewhat uncertain (see Fig 4). The slope value obviously depends on bullet design details.

The bullet is unstable with the coefficient slope like depicted in Fig 4. At some point of falling the bullet will start tumbling over. However, it will remain to make coning motion around yaw angle 133 at the steepest coefficient negative slope value (see Fig 4) and the landing speed will be only about 40 m/s.

Small or zero slope value at yaw angle 180º makes the base first falling bullet flight stable and oscillations are damped at the end of the trajectory despite some disturbances present. The landing velocity is approximately 85 m/s.

If the slope value is moderate only minor instability occurs and the bullet will land base first with high speed 85 m/s. However, for example the coupling resonance detected (ie buffeting) around the apex may evoke the fast mode oscillation which in this case leads later to large oscillations and retarded landing velocities.

The resonance does not take place below launch angle value 86º due to the bullet high velocity at the apex (flow frequency too high) and at launch angle 90º due to the aerodynamic model used (no flow oscillations present). The possible bullet/flow frequency matching time window (bullet velocity < 10 m/s) is order of 1 s around the apex and the flow oscillation (ie the Magnus-moment coefficient oscillation here) was in the simulations made to take place inside that frame. In the example of coupling at the launch angle 86º the matching was made to occur in time zone 19.9…20.0 s (lasting only 0.1 s). The bullet angle of attack is there about 90º and the flight velocity approximately 9 m/s. Figure 8 illustrates the bullet transversal angular velocity time history with and without body/flow matching frequencies. The non-matching case with 10 % increased flow frequency is at the top of Figure.

The matching frequencies caused coupling phenomenon introduces energy to the system studied at the unstable frequency (the fast mode here). The oscillation velocity is seen to be evoked shortly after the apex (Fig 8, below).

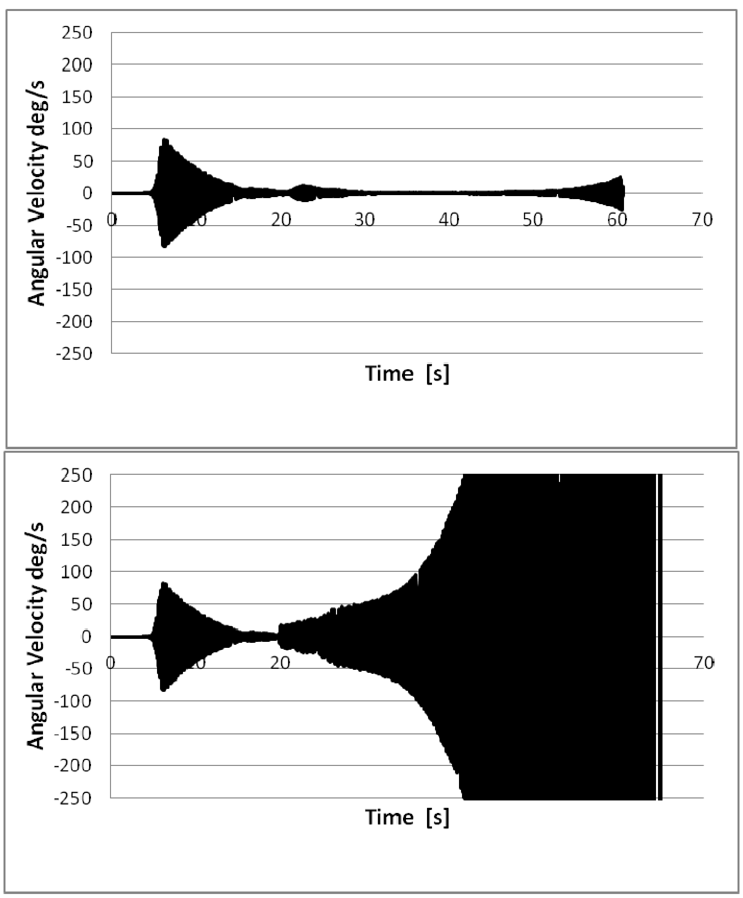

Figure 8. The bullet lateral angular velocities as a function of time.

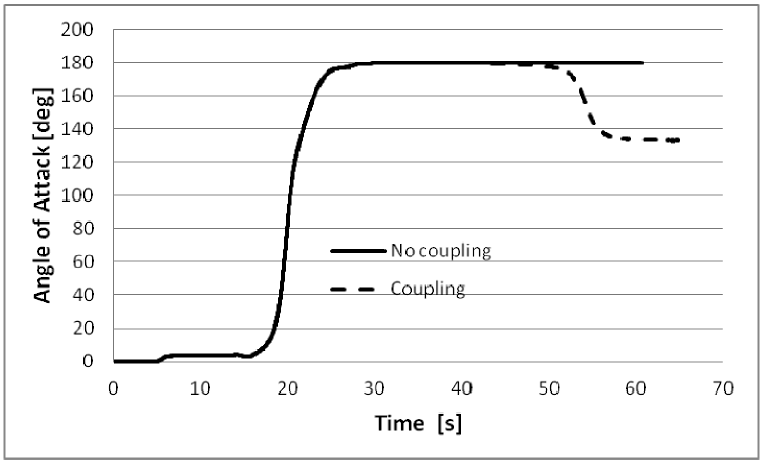

Figure 9. The bullet total angle of attack as a function of flight time with the initial trajectory launch angle 86º (with and without resonance present).

The bullet angle of attack time histories are depicted in Fig. 9. The large coning motion is seen to increase at the end of the flight in case of bullet/flow resonance near apex. The bullet will again remain to make coning motion around yaw angle 133 at the steepest Magnus-moment coefficient negative slope value (see Fig 4) and the landing speed will be only about 40 m/s.

Bullet Terminal Velocity Effect

At launch angles ≤ 80º the nose down landing bullet seems to possess at least the estimated minimum lethal energy 40 J [16] after falling down from the altitude of about 3 km. The energy 40 J corresponds now to Terminal Velocity (TV) of about 92 m/s (see also [17]).

At very large launch angles over 80º the skull penetrating speed 60 m/s [17] is clearly exceeded without velocity retarding resonance or instability present.

CONCLUSIONS

The computational study undertaken shows, that the generic military bullet terminal velocities are 100…135 m/s (max ~ 485 km/h ~ 300 mph) if the launch angle is 15º…80º. The bullet angle of attack remains clearly below 90º and the bullet flies “nose first” all the time in this region. However, the small launch angle region was not studied much in this paper and the terminal velocities/velocity reduction of nose down falling bullets is a subject of further studies.

In the launch angle region of 80º…90º the bullet basically lands the base first. The terminal velocity might vary between values 40…85 m/s. The result depends on possible Magnus-moment caused bullet instability or the bullet/flow resonance. The buffeting-like phenomenon described is new to the authors of the current paper at this particular context. However, the flow time-dependent phenomena detected were found out to have negligible effect on flight without matching of the natural frequencies (flow/bullet).

Experimental result found for an upwards fired 7.62 mm bullet terminal velocity is about 90 m/s, which is near to the base first landing case simulated result. The typical terminal velocities given in literature for spent bullets are from 300 fps to 600 fps (90...180 m/s) [17].

In many simulated cases through the launch angle region the bullet possessed the estimated minimum lethal energy 40 J at the end of trajectory. The skull penetrating speed 60 m/s was mostly clearly exceeded. A preliminary value for shooter-centered danger zone diameter obtained was found out to be approximately 8 km.

NOMENCLATURE

A	=	axial force
a	=	speed of sound
C_A	=	axial force coefficient A/qS
C_D	=	drag coefficient D/qS
C_{D_0}	=	zero yaw drag coefficient
C_L	=	lift force coefficient L/qS
C_l	=	rolling moment coefficient $\mathcal{L}/qSd$
C_{l_p}	=	spin damping moment coefficient $\partial C_l / \partial\left(pd/2V\right)$
C_m	=	overturning (pitch) moment coefficient $\mathcal{M}/qSd$
C_{m_q}	=	pitch damping moment coefficient $\partial C_m / \partial\left(Qd/2V\right)$
C_n	=	Yawing moment coefficient N/qSd
C_{n_p}	=	Magnus-moment coefficient $C_n/(pd/2V)$
C_N	=	normal force coefficient N/qS
C_{N_α}	=	normal force coefficient slope $\partial C_N/\partial\alpha$
C_Y	=	side force coefficient Y/qS
C_{Y_p}	=	Magnus-force coefficient
CG	=	center of gravity
D	=	drag
d	=	projectile diameter
f	=	frequency
I_x	=	inertia moment, longitudinal
I_y, I_z	=	inertia moment, transverse
L	=	lift force
$\mathcal{L}$	=	rolling moment
l	=	length
M	=	overturning moment, pitching moment
Ma	=	Mach number
N	=	normal force, Magnus (yawing) moment
p	=	projectile spin rate
$\hat{p}$	=	dimensionless spin or spin ratio $pd/2V$
Q	=	angular velocity

q = kinetic pressure $(1/2)\rho V^2$

Re_d = Reynolds number $\rho Vd/\mu$

S = cross section area (reference area) $\pi d^2/4$

St = Strouhal number fd/V

T = temperature

TV = terminal Velocity

V = velocity

Y = side force

α = angle of attack, total angle of attack, yaw angle

ρ = air density

μ = dynamic viscosity

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APPENDIX A: BULLET AERODYNAMIC MODEL

TABLE A1. BULLET AERODYNAMIC PROPERTIES.

Coefficient / Formulae	Limits
Zero drag force coefficient	
$C_{Do} = 0.16$	Ma<1
$C_{Do} = 0.8(0.5 - (Ma-1)/10)$	Ma>=1
Axial force coefficient	
$C_A(\alpha) = [\cos(\alpha)]^2 C_{Do} - [\sin(2\alpha)]/5$	α = 0...90 deg
$C_A(\alpha) = 0.5[\cos(\alpha)] + \sin(2\alpha)/4$	α = 90...180 deg
Pitching moment coefficient	
$C_m(\alpha) = 2.8\sin(2\alpha)/2$	α = 0…90 deg
$C_m(\alpha) = 1.4\sin(2\alpha)/2$	α = 90...180 deg
Normal force coefficient	
$C_N(\alpha) = 2\sin\alpha + 0.8(\sin\alpha)^2$	α = 0…180 deg
Magnus-moment coefficient ($pd/(2V)=1$)	
$C_{np}(\alpha) = 0.2\sin(3\alpha)$	α = 0...60 deg
$C_{np}(\alpha) = -0.9(\sin(1.5(\alpha - 60\pi/180))$	α = 60...180 deg
$-(\sin(1.5(\alpha - 60\pi/180)))^{10})$	
$C_{np} = 0.1Ma - 0.1$	@ 2.5 deg Ma<1
Magnus-force coefficient ($pd/(2V)=1$)	
$C_{Yp}(\alpha) = -3(\sin\alpha)^5$	α = 0…90 deg
$C_{Yp}(\alpha) = -3\sin\alpha$	α = 90...180 deg
Spin damping moment coefficient	
$C_{lp} = -0.035 + Ma/150$	
Pitch damping moment coefficient	
$C_{mq} = -2 - 8Ma^2$	Ma<1
$C_{mq} = -10$	Ma>=1

The often used wind-coordinate aerodynamic force-system (L, D and S in Figure 9) is replaced in this study by aero-ballistic force system (N = normal force in the x_bV-level oblique to the x_b-axis and A = axial force parallel with x_b). The aerodynamic force coefficients in the wind coordinate system can be obtained from

$$C_D(\alpha) = C_N\sin(\alpha) + C_A\cos(\alpha) \quad \text{(A1)}$$

$$C_L(\alpha) = C_N\cos(\alpha) - C_A\sin(\alpha) \quad \text{(A2)}$$

It is essential to notice that the aerodynamic model is only representative at large velocities (>150 m/s). Nevertheless, it gives reasonable time history for bullet small yaw-angle flight up to the apex. Also one must realize that the angle of attack dependence on air resistance for example is in this study taken into account via use of axial and normal force.