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ON THE CRATER DIAMETERS IN METALLIC PLATES IMPACTED BY HIGH VELOCITY RODS

Roman Kositski, Zvi Rosenberg and Erez Dekel
Rafael Ltd. Haifa, Israel

The issue of crater diameter in metallic targets, which are impacted by high velocity rods, is investigated through numerical simulations for both semi-infinite targets and finite thickness plates. We find that the predictions from two analytical models, which have been proposed for semi-infinite targets, are in very good agreement with our simulation results, enhancing the validity of these models. In addition, we construct a numerically based model which accounts for our simulation results for the crater diameter in finite thickness plates. This, rather simple model, can be used for estimating the diameters of the holes which are produced in the metallic plates of reactive armors by high velocity shaped charge jets.

INTRODUCTION

The penetration process of shaped charge jets into metallic targets has been analyzed through the hydrodynamic penetration model quite successfully for over 60 years. The effects of target strength and jet particulation have been accounted for by later refinements of the model, as reviewed by Walters et al [1]. Thus, as far as penetration depths are concerned, the predictions from these models agree with both experimental data and with numerical simulations.

The primary disruption mechanisms of shaped charge jets, by explosive reactive armors (ERA), is due to the interaction between the jet and the edges of the holes, in the perforated metallic plates, which are propelled by the explosive layer. In order to construct an analytical model for the jet/plate interaction one has to follow the evolution of the hole diameter, in a given plate, during and after it has been perforated by the jet. As a first step, one should consider the work of Szendrei [2], who presented an analytical model of crater formation in a semi-infinite target as it is penetrated by a high velocity jet. Naz [3] simplified this analysis and compared its predictions with experimental data for various steel targets impacted by high velocity copper rods. Shinar et al. [4] proposed a variation to Szendrei's model and found good agreement between their model's predictions and the experimental results of Naz [3]. In a more recent work, Partom [5] performed numerical simulations with tungsten jets impacting various steel targets, in the velocity range of 1.5-3.0km/s. These simulations resulted in larger crater diameters than those predicted by the model of Szendrei [6]. These models should be considered as the initial step in the effort to derive a comprehensive model for the crater evolution in thin plates, in order to follow the jet/plate interactions in ERA designs.

The purpose of the present paper is to further explore this issue by performing numerical simulations of high velocity rods impacting semi-infinite metallic targets. In addition, we performed numerical simulations of these high velocity rods, perforating thin plates, and followed the crater's growth process in order to have more insight into the working of reactive armors, as explained above.

ANALYTICAL MODELS

The analytic models discussed above, for the crater's growth, are derived from momentum considerations. The basic process is of a jet eroding continuously and flowing radially from its point of contact with the target. The jet exerts a high pressure on the target, transferring lateral momentum and imparting a radial velocity to the crater, which is resisted by the target's strength.

Szendrei's model follows the time history of the crater's radial growth, resulting in the following expression for the final crater radius (r_f):

$$r_f = \sqrt{\frac{\rho_p r_p^2 V_p^2}{2\sigma\left[1 + K\sqrt{\frac{\rho_p}{\rho_t}}\right]^2}} \tag{1}$$

where ρ denotes density and subscripts 'p' and 't' stand for the penetrator and target, respectively. V_p is the penetrator's velocity, σ describes the target's resistance to the radial flow, which is of the order of the target's flow stress (Y) in the models cited above.

The parameter K in Eq. (1) is related to the erosion rate of the jet, through the hydrodynamic theory:

$$\frac{dP}{dL} = K\sqrt{\frac{\rho_p}{\rho_t}} \tag{2}$$

where P and L are the penetration depth and jet length, respectively. In fact, the magnitude of K expresses the deviation from the hydrodynamics (no strength for both jet and target), in which case K=1.0. Once the strength of the target enters into consideration K should have a value which is smaller than 1.0. Naz [3] assumed that for all practical purposes $\frac{1}{2}\rho_p V_p^2 \gg Y$, so that strength terms are negligible as compared with the Bernoulli pressures, and that K should be equal to 1.0.

Naz and Woidneck [7] assumed that K=1 and that σ=Y and rewrote Eq.(1) in the form:

$$r_f = r_p V_p \sqrt{\frac{1}{2Y\left(\frac{1}{\sqrt{\rho_p}} + \frac{1}{\sqrt{\rho_t}}\right)^2}} \tag{3}$$

For jet and target of equal density this equation simplifies to:

$$r_f = r_p V_p \sqrt{\frac{\rho}{8Y}} \tag{4}$$

Shinar et al [4] based their model on the principles of dynamic plasticity. Starting from the crater equation of motion they assumed that the crater's lateral velocity is given by kU, where U is the penetration velocity as defined by:

$$U = \frac{V_p}{1 + \sqrt{\frac{\rho_t}{\rho_p}}} \tag{5}$$

They chose a value of k = $\sqrt{2}/2$ on empirical grounds, and their expression for the final crater radius is:

$$r_f = r_p \sqrt{1 + \frac{V_p^2}{2\sigma\left(\frac{1}{\sqrt{\rho_p}} + \frac{1}{\sqrt{\rho_t}}\right)^2}} \tag{6}$$

where $\sigma = 2Y/\sqrt{3}$.

It is clear that this expression is very similar to that of Szendrei for the crater's radius, Eq. (1). In fact, for large enough velocities (V_p>3km/s) the predictions from these equations are, effectively, the same. Note that the models cited above include some empirical constants, which can be determined by a few experiments or by numerical simulations, as will be shown next.

NUMERICAL SIMULATIONS

A series of two-dimensional axi-symmetric simulations were performed using ANSYS/AUTODYN multi-material Eulerian solver. A constant mesh size of 0.25mm was used which, amounts to 12 cells on a jet diameter of 3mm. Several simulations were performed with 0.5mm and 0.125 mm cells, in order to check for the convergence of simulations with 0.25 mm cells. The radial and back surfaces of the thick targets, and the radial surface of the thin plates, were set with the FLOW boundary conditions, guaranteeing that waves do not reflect from these boundaries. Fig (1) shows two snapshots from a typical simulation of a jet penetrating a semi-infinite target. The jet was modeled as a long rod (L=100mm) with constant velocity. The velocities were: 3, 5 and 7 km/s. The rod material was modeled with only compressibility but no strength.

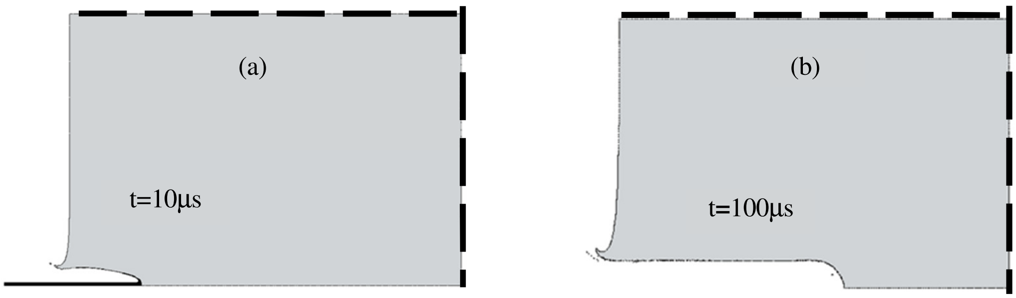

Figure 1. Snapshots from a typical simulation. (a) at 10μs after impact, (b) final crater at 100μs after impact. Note the FLOW boundary as marked by the dashed line.

For both steel and aluminum a Mie-Gruneisen equation of state was used and Table I lists the material parameters, taken from the AUTODYN material library, which we used in our simulations. A simple von-Mises strength model was chosen for the targets (Table II) while the jet had no strength, in order to represent a non-viscous fluid. The simulations were performed for target thicknesses, ranging from 2 to 60 mm. The jet was simulated as a rod (no velocity gradient), 3mm in diameter, having an aspect ratio of L/D=20.

TABLE I. Mie-Gruneisen EOS parameters.

Material	**Gruneisen Γ**	**Sound Speed [mm/μs]**	**Shock velocity slope S**
Steel	1.67	4.61	1.73
Aluminum	1.97	5.24	1.338

TABLE II. Von-Mises Strength model parameters

Material	**Density [gr/cc]**	**Shear Modulus [GPa]**
Steel	7.85	64.1
Aluminum	2.75	27

Semi-infinite targets

The first series of simulations was performed for steel rods impacting steel targets, with strengths ranging between 0.2-2.0GPa. The impact velocity in these simulations was V_p=7km/s. The crater's radius was measured at about the middle of the crater's depth, in order to avoid the influence of the entrance phase. Figure 2 shows the dependence of the normalized crater radius (divided by the jet's radius) as a function of target strength. The figure compares the results of our simulations with those predicted by the models of Szendrei [2] and Shinar et al [4]. We used K=1.0 and σ=Y for the Szendrei model, as recommended by Naz [3]. The model of Shinar et al [4] needs no fitting parameters, as seen by Eq. (5). Note that this equation was derived through some empirical considerations, as described above. One can clearly see that the general trend of the simulation results is very similar to that of the two models. The normalized crater radii in the simulations are lower by about 10% and 5%, as compared with the Szendrei and Shinar et al models, respectively.

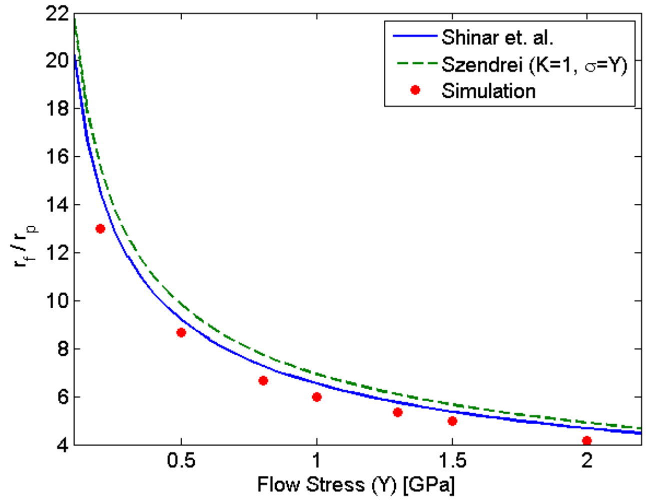

Figure 2. Normalized crater radii for semi-infinite steel targets impacted by steel rods at 7 km/s.

In order to investigate the issue of the parameter K, we determined the values of $\Delta P/\Delta L$ in several simulations, using Eq. (2). We found that K varies in the range of 0.87-0.92, depending on target strength and rod velocity. Thus, the assumption that K=1.0 is quite reasonable. Note that for the hydrodynamic case (both rod and target with no strength) the value of K should be 1.0. Thus, for strength-less rods, impacting a target with a finite strength, K should be smaller than 1.0, as we found in our simulations. It is clear that if we use K<1.0 in Eq. (1) we would get larger crater radii, which will shift up the Szendrei curve in Figure 1.

The influence of impact velocity on the crater radius was determined by a series of simulations for a steel target (Y=1.0 GPa), penetrated by a steel rod at velocities in the range of 3.0- 7.0 km/s. Figure 3 shows the results of these simulations, and the predictions from the models, for the final normalized crater radius as a function of impact velocity. Here again we find that the models over predict our simulation results by about the same amount, as for the strength dependence results shown in Figure 2.

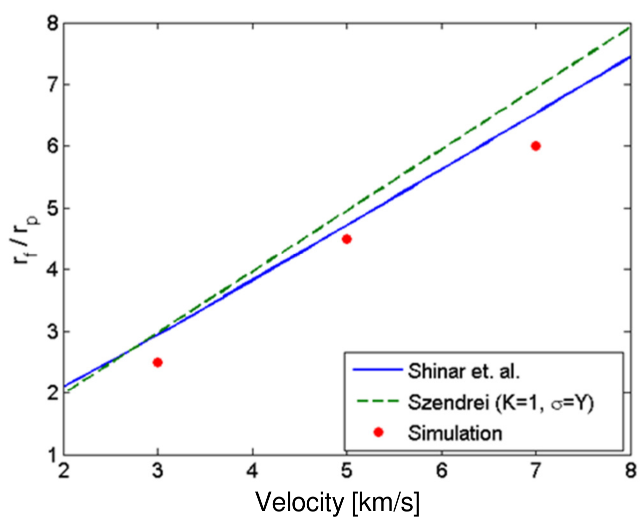

Figure 3. Final crater radius for semi-infinite steel targets with Y=1GPa impacted by steel rods.

The simulations described so far were done for steel rods penetrating steel targets. In order to investigate the influence of target density, a series of simulation was performed for steel rods penetrating aluminum targets. Figure 4 shows the result of these simulations, and we find a very good agreement with the model of Shinar et al, Eq. (6).

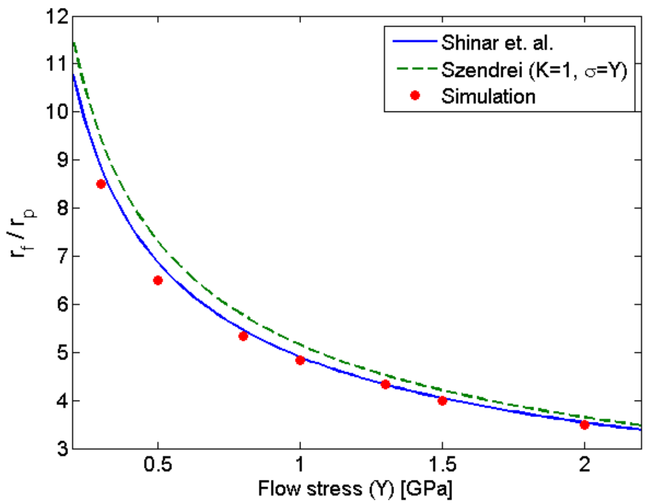

Figure 4. Normalized crater radii for semi-infinite aluminum targets impacted by steel rods at 7 km/s.

Finite targets

Simulations with finite targets were performed for various materials and different rod velocities. We found that for plates which are thicker than the rod diameter, the crater radii vary significantly between the impact face and the back face. An example of such a crater, with varying diameter, is shown in figure 5. For these plates, we determined the minimum hole radius as shown in the figure.

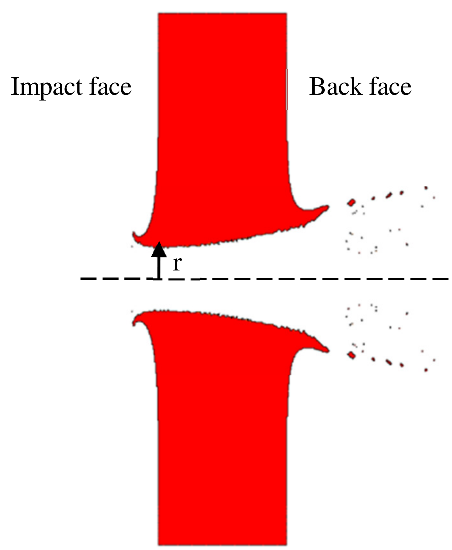

Figure 5. Perforated plate, the crater's radius was measured at the center of plate thickness.

Our next step was to determine an analytical expression for the hole radii, which will fit the numerical data for different plate thicknesses. It is clear that such an expression should asymptote to the crater's diameter in a semi-infinite target, which is designated by: $r_f/r_p=q$. In addition, for very thin plates the hole radius should be the same as the penetrator radius: $r_f/r_p=1$. The following expression applies for both requirements:

$$\frac{r_f}{r_p} = q - (q-1)e^{-\alpha\frac{h}{d}} \tag{7}$$

where h and d are the plate's thickness and the rod's diameter, respectively.

The parameter q in Eq.(7) is taken from our simulation results for semi-infinite targets, thus leaving a single "empirical" parameter (α) in our numerically- based model. By curve fitting the numerical data for the steel plates with flow strength of 1.0GPa, which are perforated by steel rods at 7 km/s, we find that α =1 for this case. It is clear that this parameter should include the dependence of the simulation results on the impact velocity of the rod as well as on the strength and density of the target. In fact, the lateral expansion of the hole is controlled by two opposing factors, the target's inertia ($\rho_t V^2$) and its strength (Y). A close examination of our simulation results, for the hole radii in different plates, led us to the conclusion that the free parameter α can be expressed as:

$$\alpha = A\sqrt{\frac{Y}{\rho_t V_p^2}} \tag{8}$$

where A is some constant.

In order to demonstrate the validity of this assumption we show in Figure 6 the agreement between the simulation results and the empirical model for steel rods, perforating steel plates (Y=1.0GPa), at three impact velocities. We start with α=1.0 for the V_p=7km/s case, as discussed above. From Eq.(8) we obtain the values of α=1.4 and α=2.33 for the 5km/s and the 3km/s cases, respectively. The figure shows the excellent agreement between our simulation results (points) and the model's predictions (curves) according to Eqs. (7) and (8).

As a further validation of our model we compare its predictions with the simulation results for the steel rods impacting steel plates with Y=0.5GPa, at 7km/s. According to Eq.(8), for this case we should use α=0.7. Again, the agreement between the model and our simulation results is very good, as is clearly shown in Figure 7.

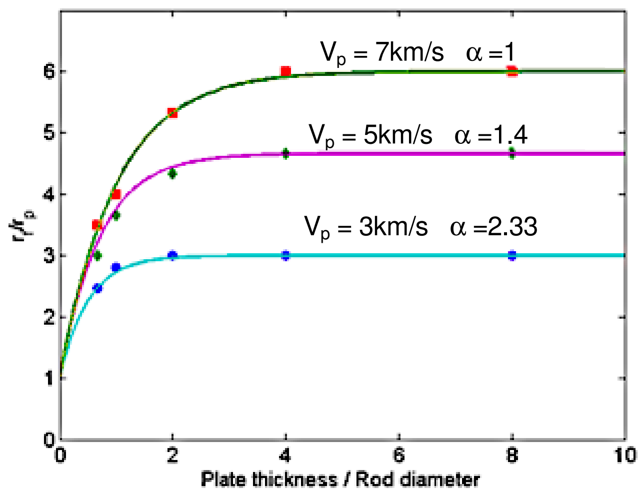

Figure 6. Normalized hole radius as a function of normalized plate thickness.

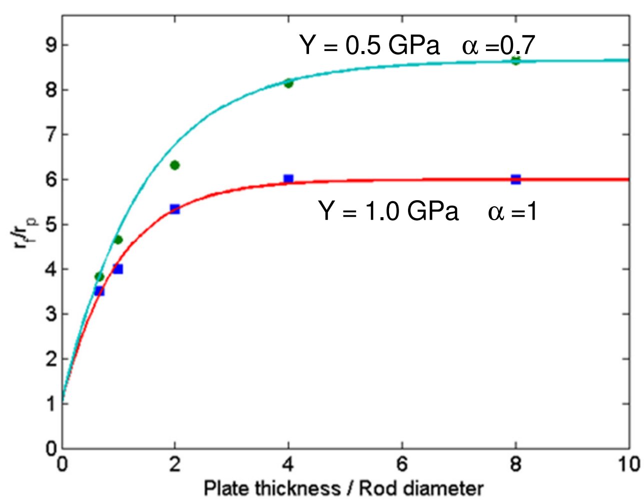

Figure 7. Different target strength. Steel targets impacted by steel rods at 7 km/s.

Finally, in order to check the validity of our model for other plate materials, we simulated the impact of 7km/s steel rods perforating aluminum plates, having a strength of Y=0.5GPa. With the reference value of α=1.0, for the Y=1.0GPa steel plates. we obtain the following value of the parameter α for this case:

$$\alpha = \sqrt{\frac{0.5}{1}}\sqrt{\frac{7.85}{2.75}} = 1.19 \tag{9}$$

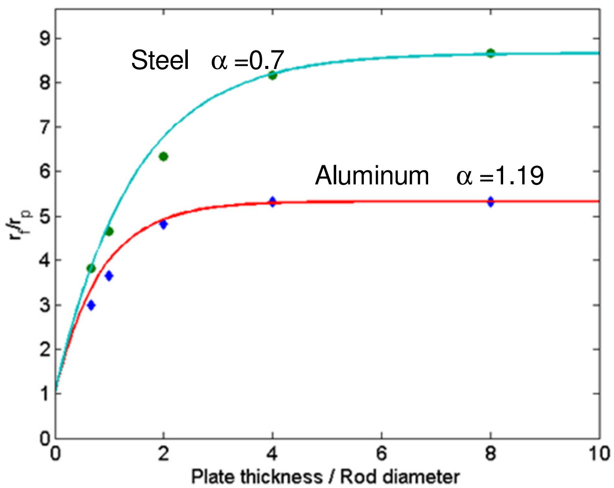

Figure 8. Steel and Aluminum plates with flow stress of 0.5GPa perforated by steel rods at 7 km/s.

Figure 8 shows the good agreement between the simulation results and the curves for steel and aluminum plates, as predicted by our model.

CONCLUSIONS

Our main conclusion, regarding the crater diameters in semi-infinite targets, is that there is a very good agreement between the predictions of the two analytical models and our simulations. The models of Szendrei [1] and Shinar et al [4] over predict the crater diameters, as compared with our simulations, by factors of about 1.1 and 1.05, respectively. We used the values of K=1.0 and σ=Y in Szendrei's model, as suggested by Naz [3]. With a somewhat higher value for the target's resistance, σ=1.25Y, this model follows almost perfectly the simulation results. The model of Shinar et al accounts for the simulation results to within 5%. This is an excellent agreement, between model and simulation, as far as analytical models in terminal ballistics are concerned. We should note that these analytical models somewhat over predict the experimental data of Naz [3] and Naz and Woidneck [7], as was pointed out Shinar et al [4]. Our simulation results agree with the experimental data from these references, enhancing the validity of these simulations.

We also performed several groups of numerical simulations for finite plates, perforated by high velocity steel rods, and found a simple formula which accounts for the normalized hole radii in these plates. With this numerically based model one can estimate the hole diameter in a thin plate through the crater's diameter in a semi-infinite target (q), and the specific value of a parameter (α) which is related to the ratio between the inertia of the target and its strength. Noting that the two analytical models for semi-infinite targets, only slightly over predict our simulation results, one can use the asymptotic values for the crater diameters (q) from these models. Our model for the hole radii in finite thickness plates is, in effect, a numerically-based "empirical" model, and one should look further into the physics of the perforation process in order to account for it. Still, as a design tool for jet/plate interactions, we believe that this simple model can account for existing data, and point to improvements with ERA designs.

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